Big Bounce Singularity of a Simple Five-Dimensional Cosmological Model

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Abstract

The big bounce singularity of a simple 5D cosmological model is studied. Contrary to the standard big bang space-time singularity, this big bounce singularity is found to be an event horizon at which the scale factor and the mass density of the universe are finite, while the pressure undergoes a sudden transition from negative infinity to positive infinity. By using coordinate transformation it is also shown that before the bounce the universe contracts deflationary. According to the proper-time, the universe may have existed for an infinitely long time.

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The inflationary cosmology can resolve three important problems of the standard big bang models: the galaxy formation problem, the horizon problem, and the flatness problem. However, there are other deep questions of cosmology which inflation does not resolve [1]: What occurred at the initial singularity? Does time exists before the big bang? These issues have been popular in cosmology for a long time. Tolman [2] firstly discussed an oscillating cosmological model within the framework of general relativity. He pointed out that the main difficulty of such an oscillating model is that the universe has to pass through a cosmological singularity on each bounce, and during each cycle, enormous inhomogeneities would undoubtedly be generated. This is the so-called entropy problem of the oscillating models. Recently, an ekpyrotic cosmological model was presented by Khoury *et. al.* within the framework of the brane world scenario [3, 4]. According to this model, our big bang universe emerges from a collision between two branes. When the two branes collide inelastically and bounce off one another, brane kinetic energy is partially converted into matter and radiation and our universe begins to expand. In the ekpyrotic model the universe undergoes a single transition from contraction to expansion [3, 4]. Drawn from his idea, Steinhardt and Turok presented a cyclic model in which the universe undergoes an endless sequence of cosmic cycles each of which begins with a "big bang" and ends with a "big crunch" [1]. It was argued that in both the ekpyrotic and cyclic model all major cosmological problems may be resolved without any use of inflation [1, 3, 4].

In this letter, we will discuss the big bounce cosmological model presented recently by Liu and Wesson [5]. This model differs from Tolman's oscillating model as well as the cyclic model in that the universe in the big bounce model undergoes a single transition from contraction to expansion. It also differs from the ekpyrotic model in that the big bounce universe contracts, before the bounce, deflationary from an empty de Sitter vacuum [5]. We will focus on a simple exact solution of the big bounce model and study what occurred at and before the bounce.

The idea of extra spatial dimensions comes from the attempt of unifying gravity with other interactions. The space-time-matter (STM) theory developed by Wesson and coworkers is inspired by the unification of matter and geometry [6, 7]. In this theory, our 4D space-time is embedded in a 5D Ricci-flat manifold, and the 4D matter fields are assumed to be "induced" from pure geometry in 5D. Mathematically, the STM theory strongly relies on Compbell's theorem which states that any solution of N-dimensional Einstein equations can

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locally be embedded in a Ricci-flat (N+1)-dimensional manifold [8]. It has been show that the STM theory agrees with all the classical tests of general relativity in the solar system [9], and it also gives physically interesting effects such as a new (fifth) force [10]. There are equivalence between STM and brane model [11]. Recently, the bounce cosmology has been used to construct brane models [12].

Within the framework of the STM theory, an exact 5D cosmological solution was given by Liu and Mashhoon in 1995 [13]. Then, in 2001, Liu and Wesson restudied the solution and showed that it describes a cosmological model with a big bounce as opposed to a big bang [5]. The 5D metric of this solution reads

$$dS^2 = B^2 dt^2 - A^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2 \tag{1}$$

where $d\Omega^2 \equiv \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$ and

$$\begin{aligned} B &= \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu} \\ A^2 &= \left(\mu^2 + k \right) y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k}. \end{aligned} \tag{2}$$

Here $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions of t, k is the 3D curvature index ($k = \pm 1, 0$), and K is a constant. This solution satisfies the 5D vacuum equations $R_{AB} = 0$. So we have

$$I_1 \equiv R = 0, \quad I_2 \equiv R^{AB} R_{AB} = 0, \quad I_3 = R_{ABCD} R^{ABCD} = \frac{72K^2}{A^8} , \tag{3}$$

which shows that K determines the curvature of the 5D manifold.

Using the 4D part of the 5D metric (1) to calculate the 4D Einstein tensor, one obtains

$$\begin{aligned} {}^{(4)}G_0^0 &= \frac{3 \left(\mu^2 + k \right)}{A^2} , \\ {}^{(4)}G_1^1 &= {}^{(4)}G_2^2 = {}^{(4)}G_3^3 = \frac{2\mu\dot{\mu}}{A\dot{A}} + \frac{\mu^2 + k}{A^2}. \end{aligned} \tag{4}$$

Suppose the induced matter is a perfect fluid with density ρ and pressure p moving with a 4-velocity $u^\alpha \equiv dx^\alpha/ds$, i.e.,

$$ {}^{(4)}T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta - p g_{\alpha\beta}. \tag{5}$$

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So $u^\alpha = (u^0, 0, 0, 0)$ and $u^0 u_0 = 1$. Substituting (4) and (5) into the 4D Einstein equations $^{(4)}G_{\alpha\beta} = {}^{(4)}T_{\alpha\beta}$, we find that

$$\rho = \frac{3\left(\mu^2 + k\right)}{A^2},$$
$$p = -\frac{2\mu\dot{\mu}}{A\dot{A}} - \frac{\mu^2 + k}{A^2}. \tag{6}$$

The solutions given in equations (1)-(6) contain two arbitrary functions $\mu(t)$ and $\nu(t)$ and are, therefore, quite general. As soon as the two functions $\mu(t)$ and $\nu(t)$ are given, the solutions are fixed. In another hand, if the coordinate t and the equation of state $p = p(\rho)$ are fixed, we can also fix the solution. In this letter, to illustrate the bounce properties explicitly, we will use the former to fix the solution. That is, we let

$$k = 0, \quad K = 1,$$
$$\nu(t) = t_c / t, \quad \mu(t) = t^{-1/2}, \tag{7}$$

where t_c is a constant. Substituting (7) into (2) and (6), we have

$$A^2 = t\left[1 + \left((y + t_c)/t\right)^2\right]$$
$$B^2 = \frac{1}{4}\left[1 - \left((y + t_c)/t\right)^2\right]^2 \left[1 + \left((y + t_c)/t\right)^2\right]^{-1} \tag{8}$$

and

$$\rho = \frac{3}{t^2\left[1 + \left((y + t_c)/t\right)^2\right]}$$
$$p = \frac{2}{t^2\left[1 - \left((y + t_c)/t\right)^2\right]} - \frac{1}{t^2\left[1 + \left((y + t_c)/t\right)^2\right]}. \tag{9}$$

Equations (8) and (9) constitute a simple exact solution. From (8) we can show that in a given $y = constant$ hypersurface the scale factor $A(t, y)$ has a minimum point at

$$t = |y + t_c| \equiv t_b, \tag{10}$$

at which we have

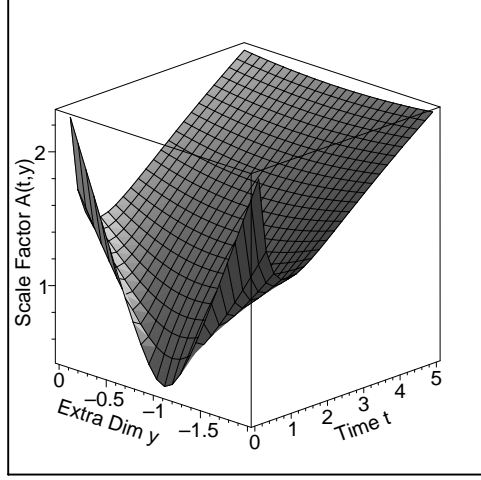$$A\big|_{t=t_b} = (2t_b)^{1/2}, \quad B\big|_{t=t_b} = 0, \quad \dot{A}\big|_{t=t_b} = 0. \tag{11}$$

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FIG. 1: Evolution of the scale factor $A(t, y) = \sqrt{t\left[1 + ((y + t_c)/t)^2\right]}$ with $t_c = 1$.

So at the bounce point $t = t_b$ the three invariants in equation (3) are normal. It means that there is no space-time singularity in the big bounce model. In equation (9), we can see that at the bounce point $t = t_b$ the pressure undergoes a transition from negative infinity to positive infinity, which corresponds to a phase transition of the matter, i.e., a matter singularity. For a radially moving photon we have $ds^2 = 0$, so $(dr/dt)|_{t=t_b} = 0$. This implies that $B = 0$ corresponds to an event horizon. For illustration, we plot the 3D graph of the evolution of the scale factor $A(t, y)$ in Figure 1. From Figure 1 we see that according to the t-coordinate, the universe evolves smoothly across its minimum at $t = t_b$. This strongly suggests that time, and the arrow of time, exist before the big bounce. However, we notice that t is not the proper-time. To make sure, we need a coordinate transformation from t to the proper-time τ. Now we let $t = t_b = |y + t_c|$ corresponds to $\tau = 0$, and we let the arrow of the τ-coordinate points in the same direction as the t-coordinate. Then from (1) and (8), the coordinate transformation reads

$$\int_0^\tau d\tau = \int_{t_b}^t |B| \, dt = \frac{1}{2} \int_{t_b}^t \left(\left|1 - (t_b/t)^2\right| \cdot \left(1 + (t_b/t)^2\right)^{-1/2}\right) dt. \tag{12}$$

The integration of (12) gives

$$\tau(t) = \frac{1}{2} \times \begin{cases} -\sqrt{t^2 + t_b^2} + t_b \ln\left(\frac{t}{t_b + \sqrt{t^2 + t_b^2}}\right) + C \;, & \text{for } 0 < t \leqslant t_b \\[2mm] \sqrt{t^2 + t_b^2} - t_b \ln\left(\frac{t}{t_b + \sqrt{t^2 + t_b^2}}\right) - C \;, & \text{for } t_b \leq t \leq +\infty \end{cases} , \tag{13}$$
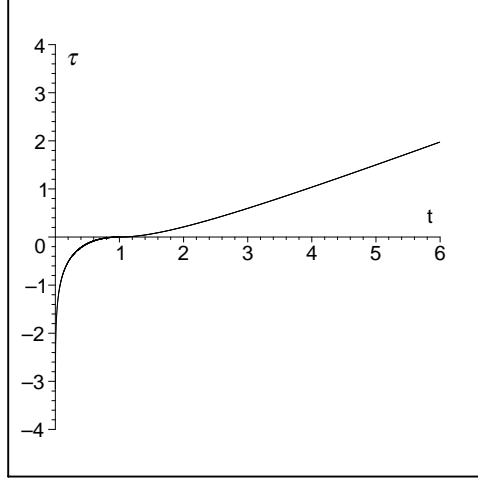
where

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FIG. 2: Coordinate transformation between the coordinate-time t and the proper-time τ.

$$C = t_b \left[\sqrt{2} - \ln \left(\sqrt{2} - 1 \right) \right]. \tag{14}$$

In the coordinate transformation (13), we find that there is an one-to-one correspondence between t and τ, and as t varies from zero to infinity, the proper-time τ varies from negative infinity to positive infinity. We also find that

$$\lim_{t \to t_b^-} \frac{d\tau}{dt} = \lim_{t \to t_b^+} \frac{d\tau}{dt} = 0. \tag{15}$$

It means that the proper-time joints together in a smooth way at the bounce point. The transformation (13) is shown in Figure 2 in which we have set $t_b = 1$ without loss of generality.   Now we discuss the evolution of the scale factor A versus the proper-time τ. For simplicity, we consider it in an approximate way as follows. For $0 < t \ll t_b$ (corresponding to $-\infty < \tau \ll 0$), we keep only the leading term in (13), so we get

$$2\tau \sim t_b \ln t$$
$$A \sim t_b t^{-\frac{1}{2}} \sim t_b e^{-\tau/t_b} \tag{16}$$

Now the 5D line element of (1) reads

$$dS^2 \approx d\tau^2 - t_b^2 e^{-2\tau/t_b} \left(dr^2 + r^2 d\Omega^2 \right) - dy^2 . \tag{17}$$

The 4D part of equation (17) is in fact the de Sitter metric, which would be interpreted as having $\rho = 0$ and $\Lambda = 3/t_b^2$. In equation (17), the scale factor is an exponential function

of proper-time τ and corresponds to a deflationary stage of the universe. In equation (8), let $t \to \infty$ (corresponding to $\tau \to \infty$), then $A \to t^{1/2}$, and $B \to 1$, the universe expands as in the standard Friedmann-Robertson-Walker (FRW) model for the radiation dominated epoch. At $t = t_b$, the scale factor reaches to its minimum point at $A = (2t_b)^{1/2}$ which corresponds to the bounce point. Using (8), (9) and (12) we can show that

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\begin{aligned}
\lim_{\tau \to 0^-} \frac{dA}{d\tau} &= -\frac{1}{\sqrt{t_b}}, \quad \lim_{\tau \to 0^+} \frac{dA}{d\tau} = \frac{1}{\sqrt{t_b}}, \\
\lim_{\tau \to 0^-} p &= -\infty, \quad \lim_{\tau \to 0^+} p = +\infty .
\end{aligned}
\tag{18}
$$

So the scale factor A expressed in terms of the proper-time τ is continuous but not smooth at bounce point. Meanwhile, the pressure undergoes a transition from negative infinity to positive infinity. This implies that a matter singularity exists at the bounce point. When $t \to 0$, $\tau \to -\infty$, which means that according to the proper-time τ, the universe has existed for an infinitely long time. Consequently, with the time elapsing in the range $(0, t_b)$, the universe contracts and crunches driven by negative pressure. At $t = t_b$ the universe gets to its minimum point, and then bounces off driven by positive pressure with radiation and matter creation. At that time the universe has a finite density $\rho = 3/(2t_b^2)$. From t_b to now, the universe expands. In summary, the bounce singularity of a simple 5D cosmological model is studied. We point out that the bounce singularity with $A \neq 0$ and $B \equiv \dot{A}/\mu = 0$ is not a space-time singularity. It is just a phase transition from de Sitter space to a FRW space. At the bounce point, the scale factor A is continuous but not smooth with respect to the proper-time τ, and the pressure has a jump from negative infinity to positive infinity which corresponds to a matter singularity and may represent a phase transition as in the inflationary models [14]. We point out that the $B = 0$ singularity is an event horizon as is in the Schwarzschild solution. According to the proper-time, the whole bounce scenario can be described as follows. Our universe has been existed for an infinitely long time. Before the bounce, the universe contracts deflationary. When it approaches to the bounce point, it undergoes a crunch driven by an infinite negative pressure, and then it bounces off driven by an infinite positive pressure and accompanied by creation of radiation and matter. After the bounce, the universe expands asymptotically as is in the standard FRW models.

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[1] P. J. Steinhardt, N. Turok, Science, **296** (2002) 1436, hep-th/0111030.

[2] R. C. Tolman, *Relativity, Thermodynamics and cosmology*, (Oxford U. Press, Clarendon Press, 1934).

[3] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64** (2001) 123522, hep-th/0103239.

[4] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and Turok, Phys. Rev. D **65** (2002) 086007, hep-th/0108187.

[5] H. Y. Liu and P.S.Wesson, Astrophys. J. **562** (2001) 1, gr-qc/0107093.

[6] P. S. Wesson, *Space-Time-Matter* (World Scentific Publishing Co. Pre. Ltd. 1999).

[7] J. M. Overduin and P. S. Wesson, Phys. Reports **283** (1997) 303.

[8] J. E. Campbell, *A Course of Differential Geometry*, (Clarendon Oxford, 1926).

[9] D. Kalligas, P. S. Wesson, C. W. F. Everitt, Astrophys. J. **439** (1995) 548; H.Y. Liu and J. M. Overduin, Astrophys. J. **538** (2000) 386.

[10] B. Mashhoon P. S. Wesson and H. Y. Liu, Gen. Rel. Gra. **30** (1998) 555; P. S. Wesson, B. Mashoon H.Y. Liu and Sajko M. N., Phys. Lett. B **456** (1999) 34; H. Y. Liu, B. Mashhoon, Phy. Lett. A **272** (2000) 26.

[11] J. Ponce de Leon, Mod. Phys. Lett. A **16** (2001) 2291-2304, gr-qc/0111011.

[12] H.Y. Liu, Phys. Lett. B **560** (2003) 149-154, hep-th/0206198.

[13] H. Y. Liu and B. Mashhoon, Ann. Phys. **4** (1995) 565.

[14] L. Kofman, A. Linde, V. Mukhanov JHEP **0210** (2002) 057, hep-th/0206088; Li X. Z., Liu D. J., Hao J. G., Chin. Phys. Lett. **20** (2) (2003) 192; Zhang Y., Chin. Phys. Lett. **19** (10) (2002) 1569; Yang G. H., Jiang Y., Duan Y. S., Chin. Phys. Lett. **18** (5) (2001) 631.